Deformation of disordered materials: a statistical physics problem?

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Deformation and flow of amorphous solids: an updated review of mesoscale elastoplastic models

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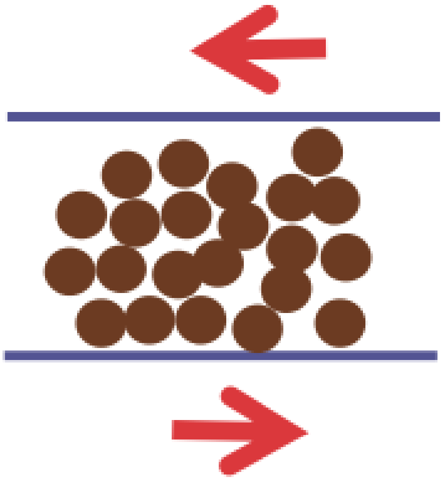
Outline

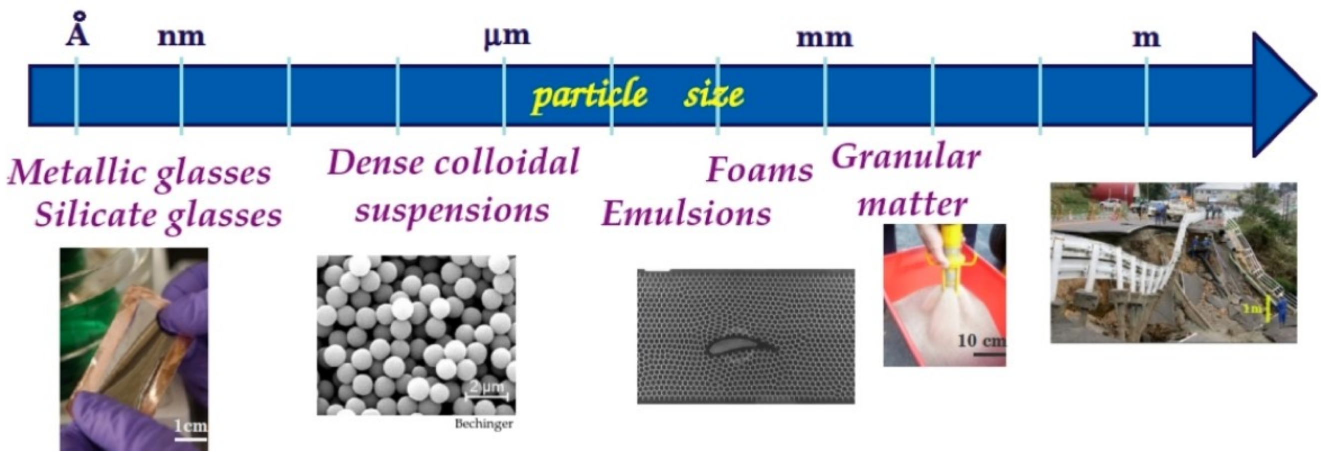
- Flow of (soft) amorphous solids
- Elastoplastic models of flow
- Mean field analysis
- Second order transition -Avalanches
- First order transition –Strain localisation
- Transients - Creep



Amorphous materials

(Yield stress fluids/soft glasses)

...spanning many orders of magnitude in terms of particle size...

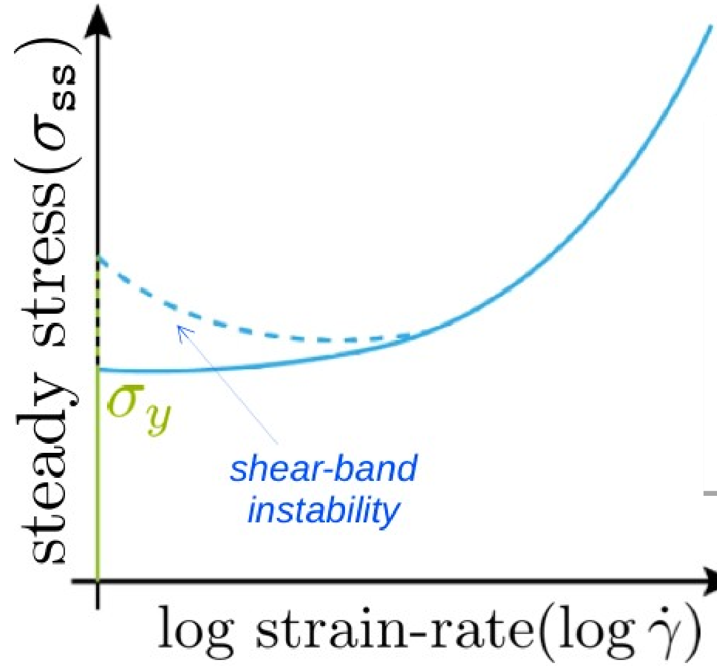


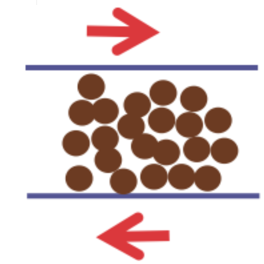
- **Disordered elastic solids, far below/above any « glass transition temperature/density**
- **Extremely diverse in terms of scales (nm-cm) and strength (100Pa-100GPa).**
- **Still share common features in their deformation mechanisms**

Non-linear rheology and yield stress:

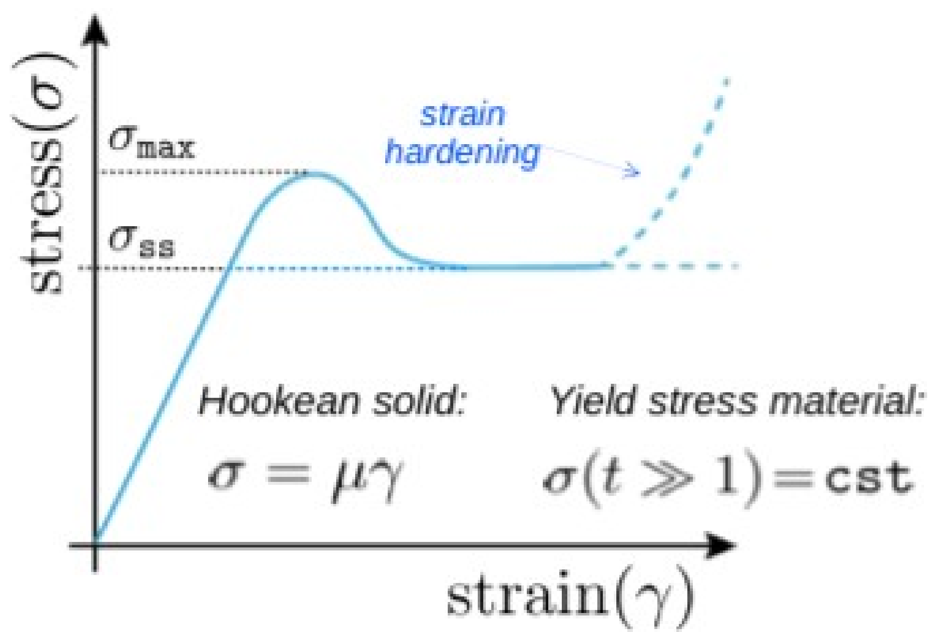
$$\sigma = \sigma_Y + A\dot{\gamma}^{1/\beta}$$

Herschel Bulkley equation



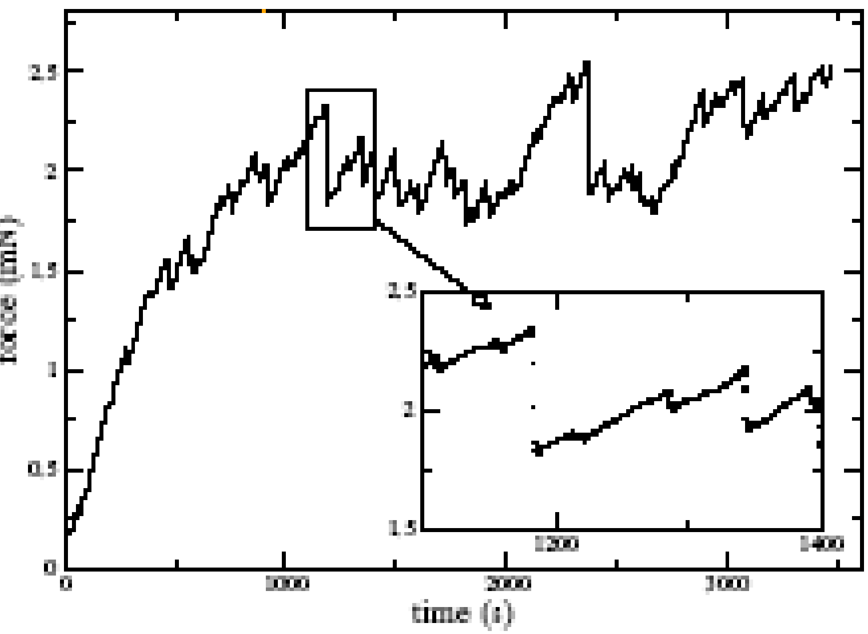


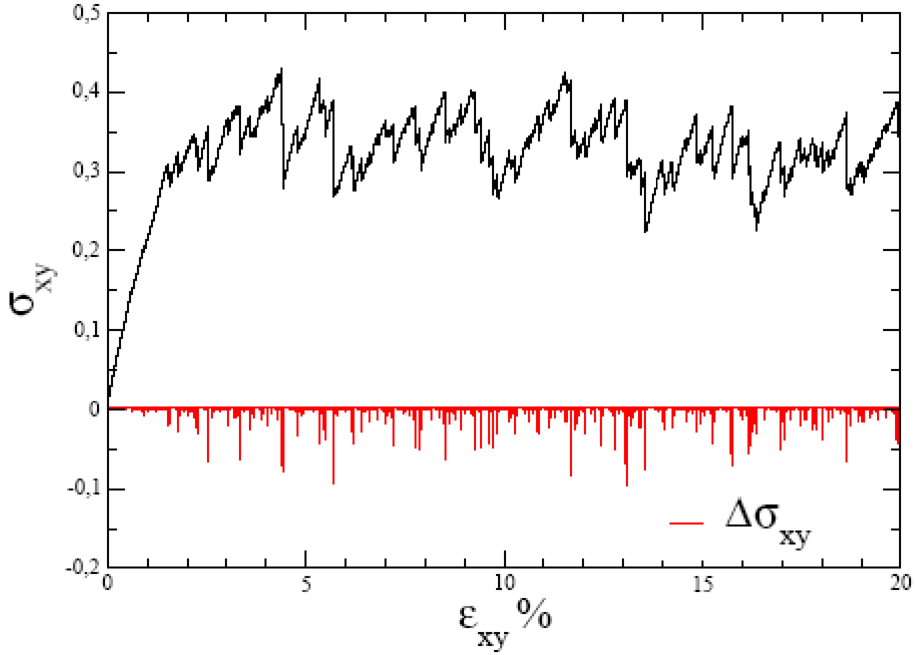
Stress-strain curves at fixed rate



Deformation at low T proceeds trough well identified *plastic events* or *shear transformations* (Argon and Kuo, 1976)

Stress-strain curve at low strain rate, low temperature, small systems



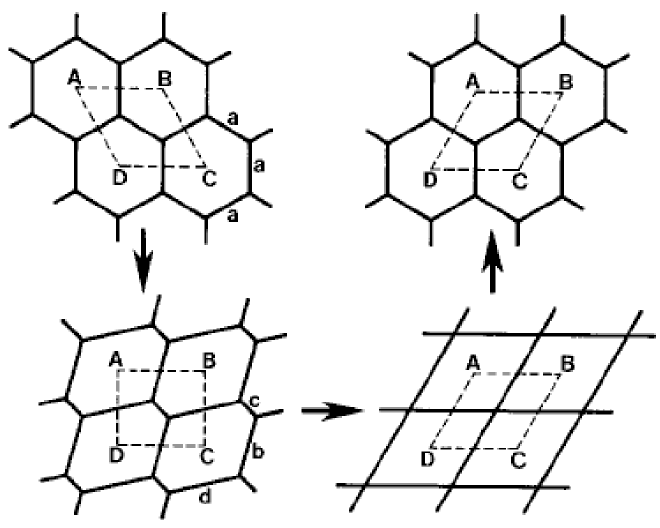


Plastic response of a foam (I. Cantat, O. Pitois, Phys. of fluids 2006)

Plastic response of a simulated Lennard-Jones glass (Tanguy, Leonforte, JLB, EPJ E 2006)

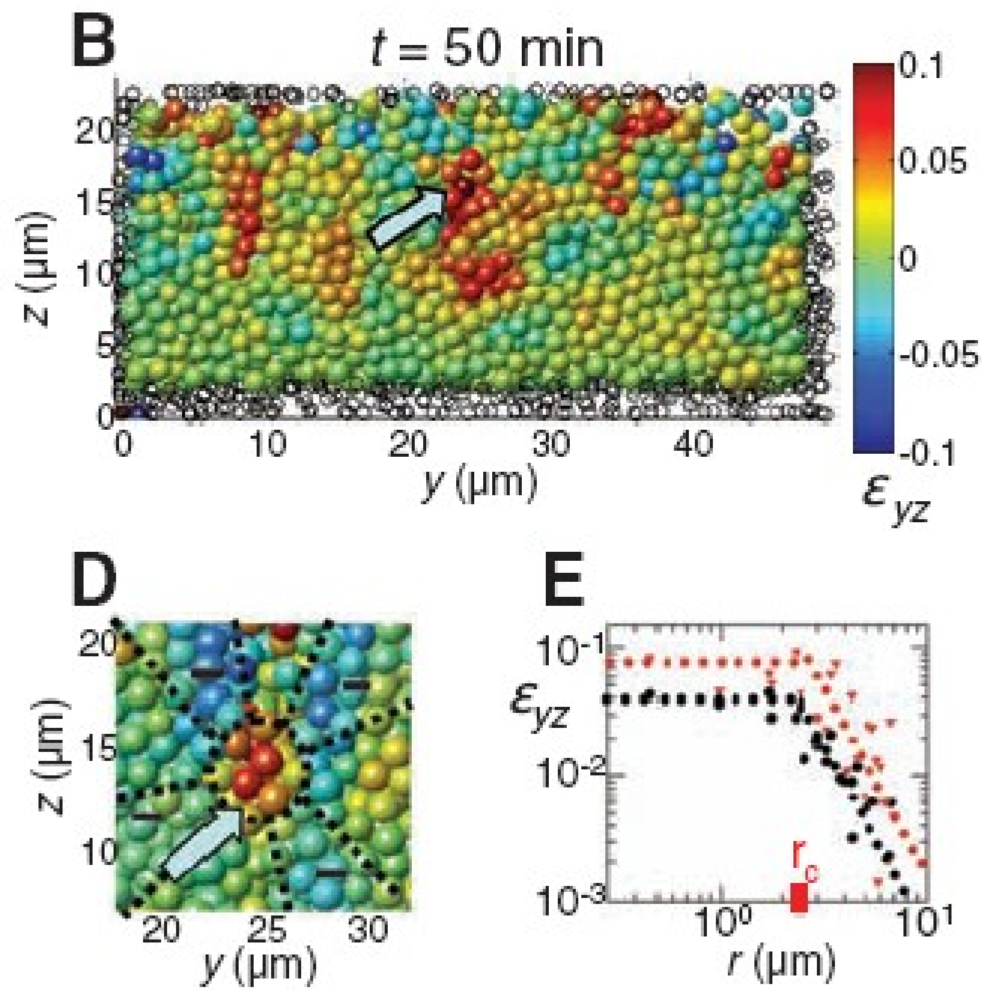
Prototypical example : « T1 » events in foams

(Princen, 1981) (Dennin 2006)



Structural Rearrangements That Govern Flow in Colloidal Glasses

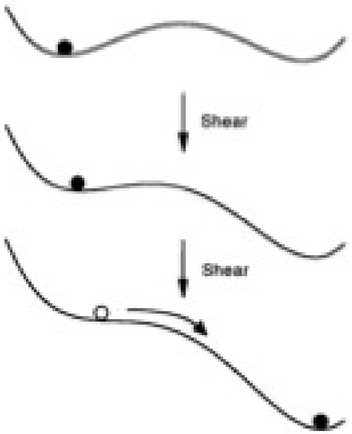
Peter Schall,[1,2*] David A. Weitz,[2,3] Frans Spaepen[2]



Colloidal glass, confocal microscopy Science 318 (2007)

Events are **shear transformations** of Eshelby type

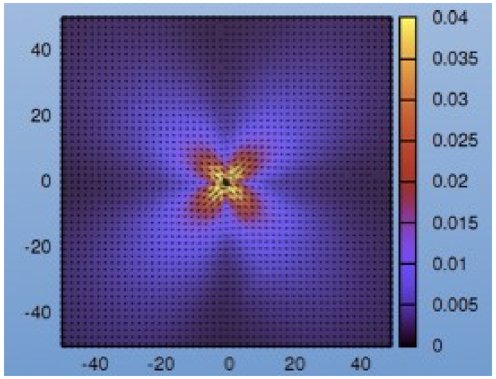
- Plastic instability in a very local region of the medium (irreversible) under the influence of the local stress.



Malandro, Lacks, PRL 1998

- Instability involves typically a few tens of particles and small shear strains (1 to 10%)

- Surroundings respond essentially as an homogeneous elastic medium (incompressible). Quadrupolar symmetry of the response.



Puosi, Rottler, JLB, PRE 2014

Events are **shear transformations** of Eshelby type

Proc. R. Soc. Lond. A 1957 **241**, 376-396

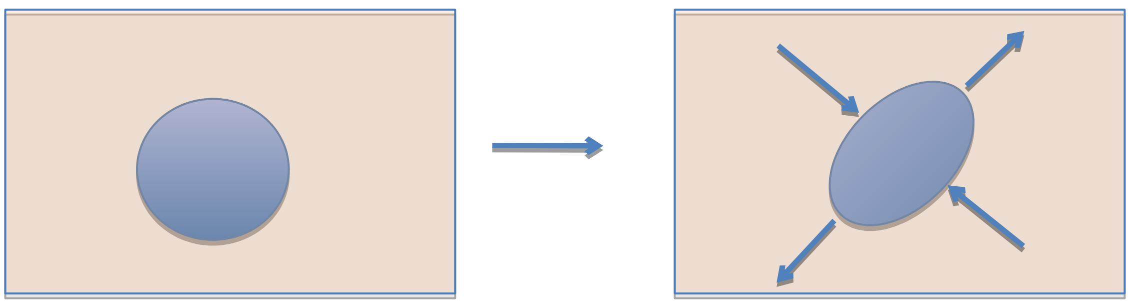
The determination of the elastic field of an ellipsoidal inclusion, and related problems

By J. D. Eshelby

Department of Physical Metallurgy, University of Birmingham

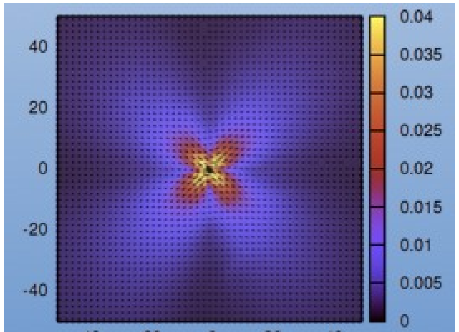
(*Communicated by R. E. Peierls, F.R.S.—Received* 1 *March* 1957)

Eshelby transformation: an inclusion within an elastic material undergoes a spontaneous change of shape (eigenstrain): circular to elliptical.



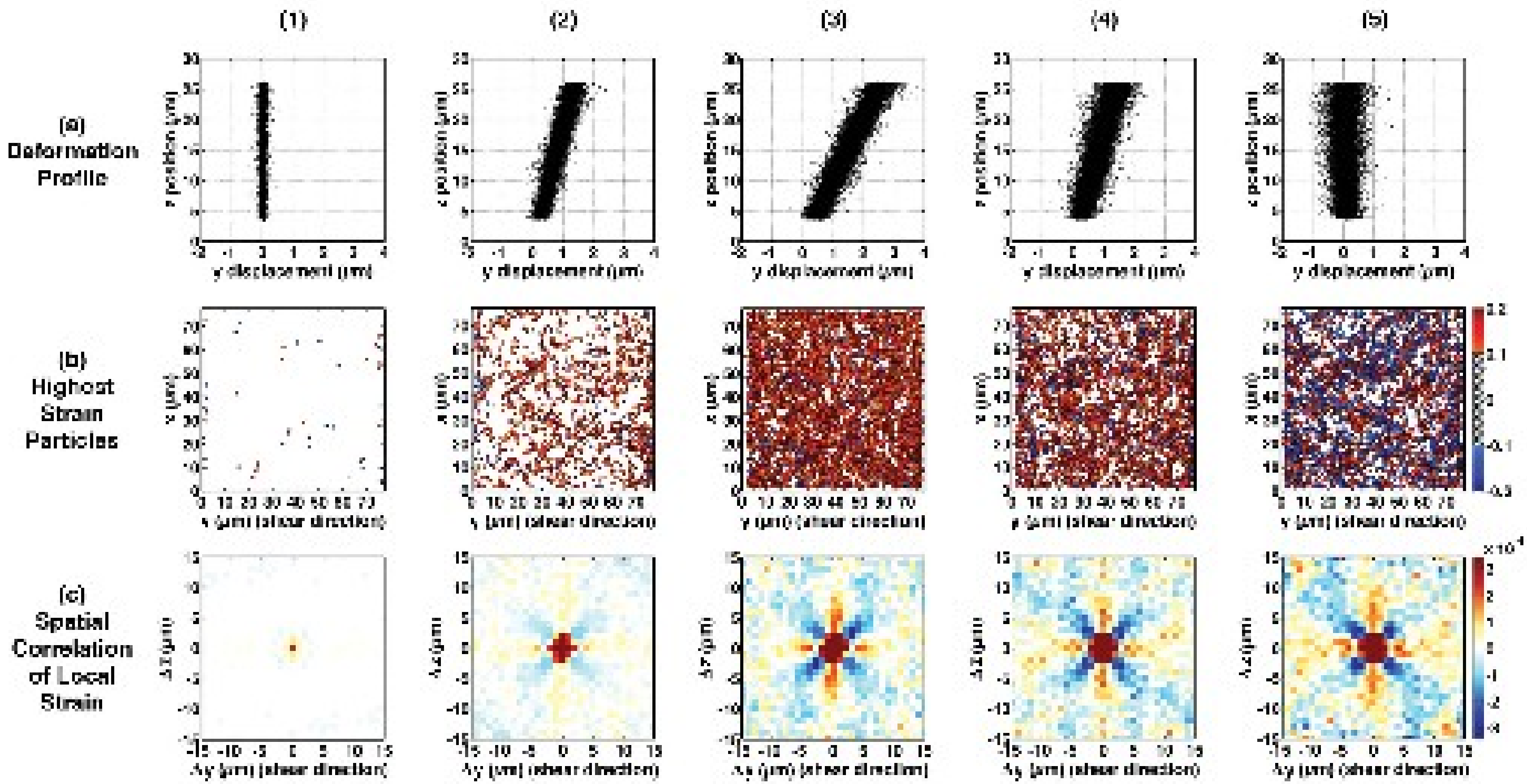
In an homeogeneous, linear elastic solid, the Induced shear stress outside the inclusion is proportional to the inclusion transformation strain and to the Eshelby propagator (response to two force dipoles):

$$G(r,\theta) = \frac{1}{\pi r^2} cos(4\theta)$$



Events are shear transformations of Eshelby type

Best seen in experiments trough correlation patterns

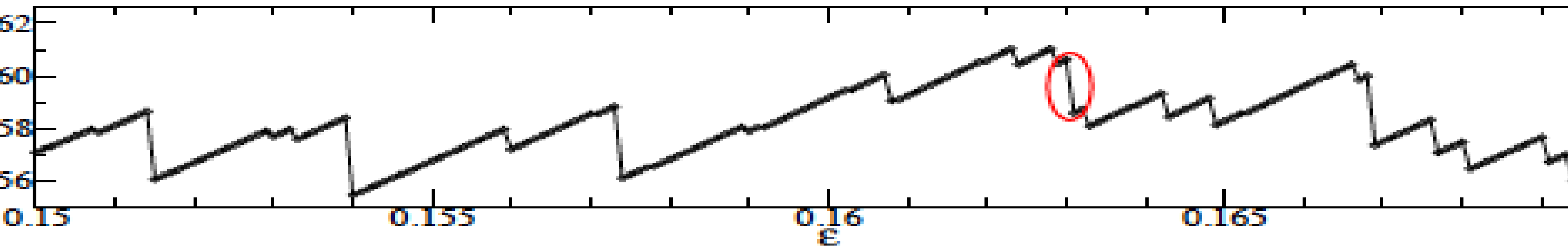


Colloidal paste under simple shear
(Jensen, Weitz, Spaepen, PRE 2014)

Stationnary plastic flow regime:

At large deformation, individual flow events interact and organize in the form of « avalanches » with a broad distribution of amplitudes.

« Barkhausen » type behavior, encountered in many condensed matter systems with disorder (magnetic domain walls, contact lines, vortices in superconductors, earthquakes, friction -> **elastic line pinned by external potential, « depinning » problem**)



Flow of amorphous solids:

- Statistical physics problem: elementary events identified, space time interactions and correlations between events lead to avalanches and noise.
- Jamming/Yielding (from flowing suspension to a solid paste) has features of a dynamical phase transition (depinning ?)

$$\sigma = \sigma_Y + A\dot{\gamma}^{1/\beta} \longleftrightarrow \dot{\gamma} \propto (\sigma - \sigma_{\text{yield}})^{\beta}$$

- Universal features from granular media to metallic glasses
- Different (simpler ?) from extensively studied crystalline plasticity which involves topological flow defects (dislocations).

=> Gain qualitative understanding of phenomena at different scales with simple models, and tune model parameters for quantitative predictions.

Three levels of modelling

- Microscopic : Particle based, molecular dynamics or athermal quasistatic deformations. Detailed information, limited sizes /times.
- Mesoscopic : Coarse grain and use the « shear transformations » as elementary events, with elastic interactions between them.
- Continuum : Stress, strain rate, and other state variables (« effective temperature ») treated as continuum fields.

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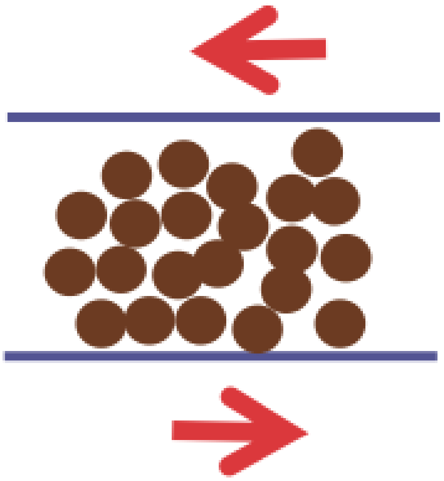
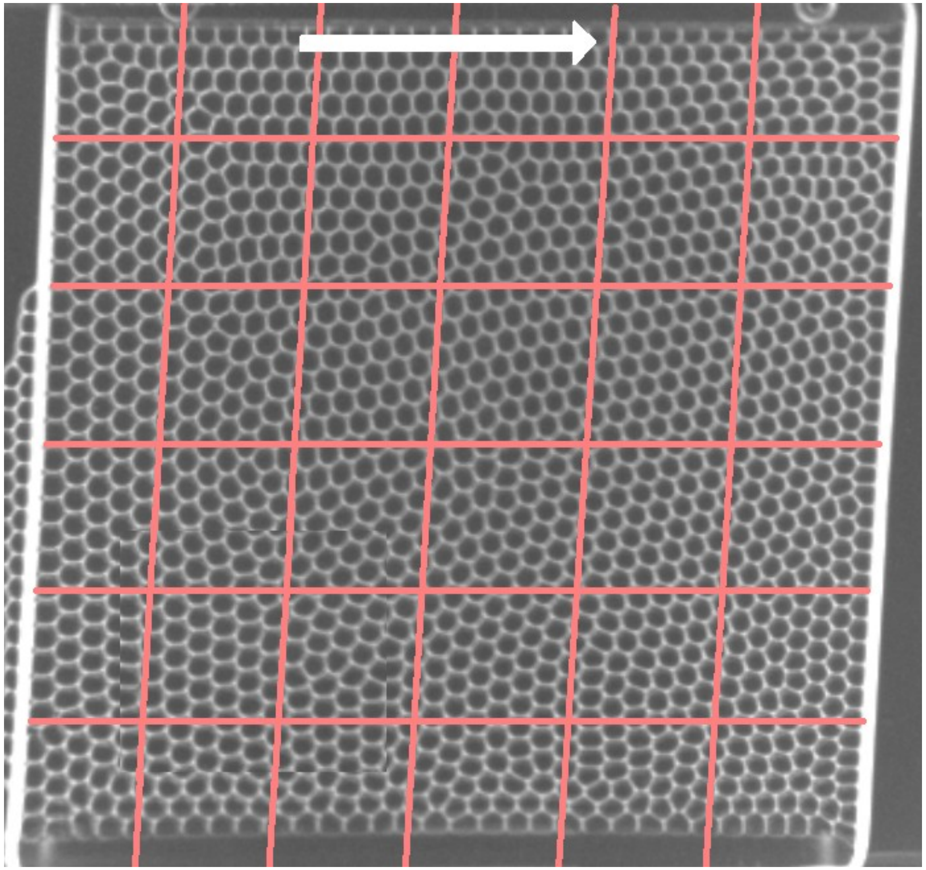


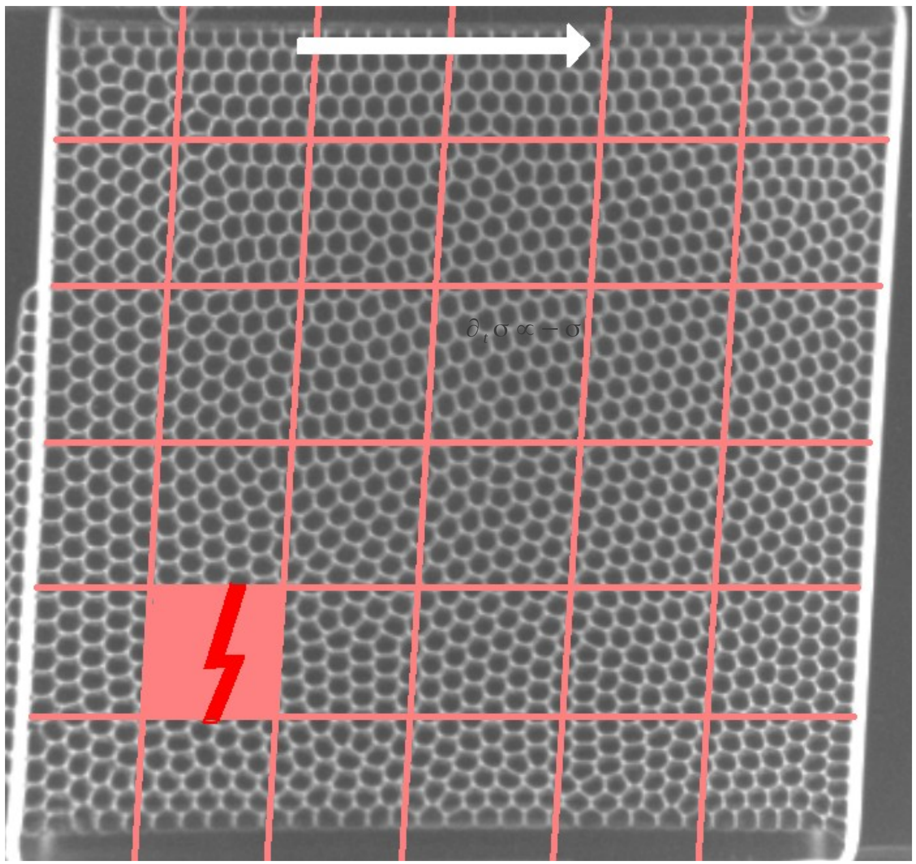
Mesoscopic elastoplastic model: « Ising model » of amorphous deformation:



Linear elastic response at low shear

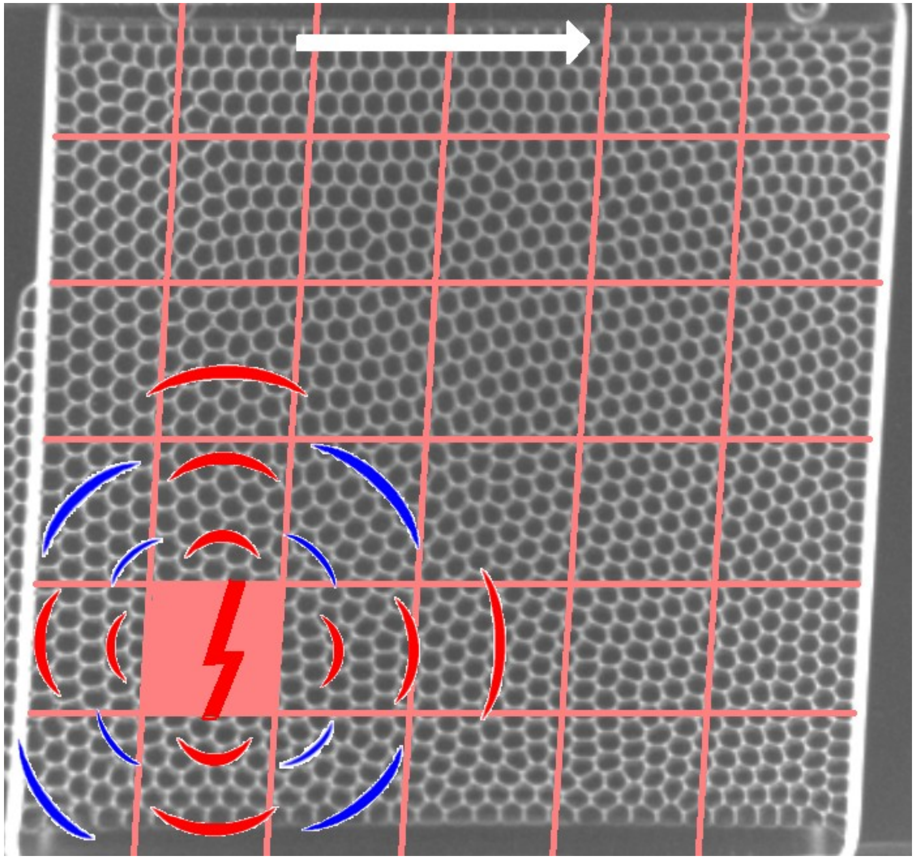
Related models by Bulatov and Argon, Picard et al

At some point (yield criterion), the material **yields** locally and the energy stored locally is dissipated → plastic event



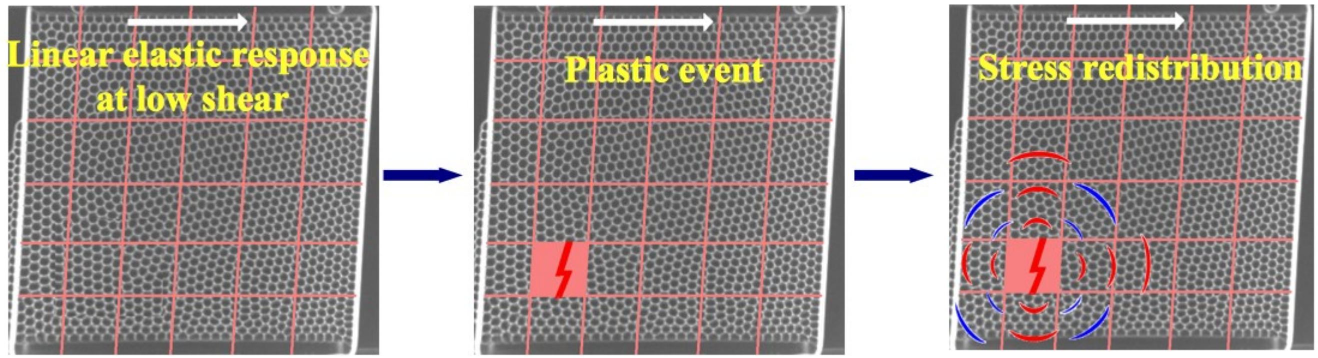
Related models by Bulatov and Argon, Picard et al

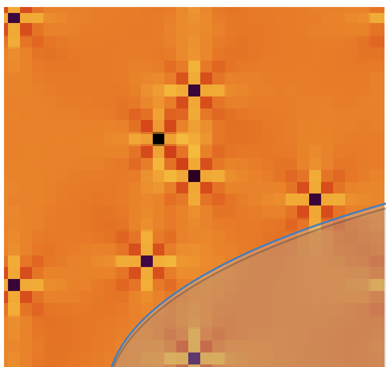
Stress is redistributed during plastic event, owing to the presence of an elastic medium



Then elastic regime again, etc.

Implementation on a grid ; σ_i local stress variable





$$\partial_t \sigma_i = \mu \left(\dot\gamma + \sum_j G_{ij} \dot\gamma^{(j)}_{plastic} \right)$$

$$G_{ij} \sim \frac{\cos(4\theta_{ij})}{r_{ij}^d}$$

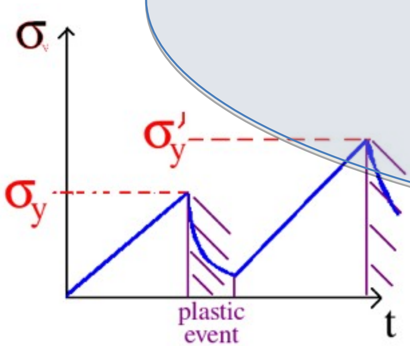
Universal

$$\dot\gamma^{(j)}_{plastic} = 0 \text{ if } \sigma_j < \sigma_Y \text{ (elastic loading)}$$

$$\dot\gamma^{(j)}_{plastic} = \mu\sigma_j/\tau \text{ if } \sigma_j < \sigma_Y \text{ (plastic activity)}$$

Site becomes elastic again after a typical plastic deformation γ_c.

Non universal



Time evolution of the local stress

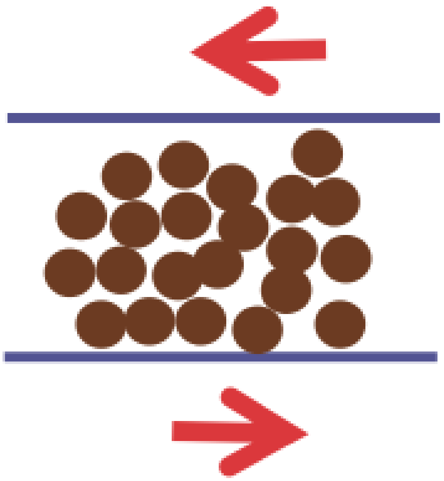
Mesoscopic description: « Ising model » of plastic deformation, implemented on a lattice; σ_i local stress variable

- Can be implemented in a finite element code rather than assuming Eshelby propagator. Allows for disorder/weakening (K. Karimi, LJB, PRE 2016) or various boundary conditions (Budrikis, Zapperi, Nat. Comm. 2017)

- Fully tensorial description and taking convection into account are possible (A. Nicolas, JLB, PRL2014). Validated against experiments (Goyon et al, flow in microchannels).

- Parameters can be related to microscopic description (Patinet, Falk, Vandembroucq ; Tanguy, Albaret)

Outline



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Mean field analysis (Hébraud Lequeux model): Stress diffusion due to mechanical noise + self consistency

P(σ,t) probability distribution of stress on **a typical site** (no disorder, single local yield stress)

$$\partial_t \mathcal{P}(\sigma,t) = -G_0\dot{\gamma}(t)\,\partial_\sigma \mathcal{P} + D_{\rm HL}(t)\,\partial_\sigma^2 \mathcal{P} - \nu_{\rm HL}(\sigma,\sigma_c)\,\mathcal{P} + \Gamma(t)\,\delta(\sigma)$$

External drive

Stress diffusion

Yield if $\sigma>\sigma_c$

Reset to zero after yield

Yield rule and plastic activity

$$\nu_{\rm HL}(\sigma,\sigma_c) \equiv \frac{1}{\tau}\theta(\sigma-\sigma_c)$$

$$\Gamma(t) = \frac{1}{\tau}\int_{\sigma'>\sigma_c} d\sigma'\,\mathcal{P}(\sigma',t)$$

Non linear feedback

$$D_{\rm HL}(t) = \alpha\,\Gamma(t)$$

Mean field analysis (Hébraud Lequeux model): Stress diffusion due to mechanical noise + self consistency

- $\alpha > \alpha_c = 2$ Newtonian behaviour
  $\sigma \sim \dot{\gamma}$

- $\alpha < \alpha_c = 2$ Herschel Bulkley law with exponent 1/2:
  $\sigma = \sigma_Y + A\dot{\gamma}^{1/2}$

A spatial version (inhomogeneous stress or strain rate) can be derived (Bocquet et al, PRL 2012) and compares well with experiments in confined geometries (A. Nicolas, JLB, PRL2014)

Disorder does not modify exponents (Agoritsas et al, 2016)

Mathematical study: Julien Olivier, **Z. Angew. Math. Phys.**, 61(3), 2010, pp 445-466

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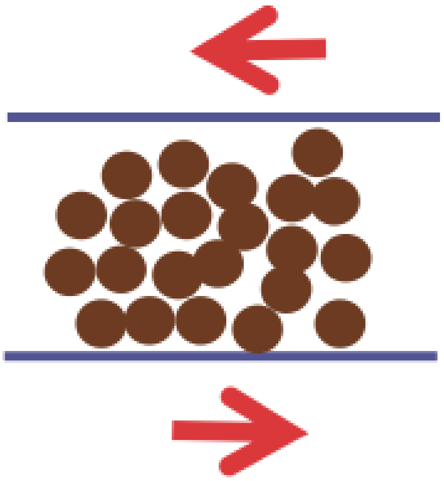


Yield (arrested->flow) transition as a second order dynamical transition

Rewrite Herschel Bulkley equation as:

$$\dot{\gamma} \propto (\sigma - \sigma_{\mathrm{yield}})^{\beta}$$

Dynamical transition with "Second order" critical behaviour, monotonous flow curve. Avalanche behavior at vanishing strain rates, analogy with depinning problems.

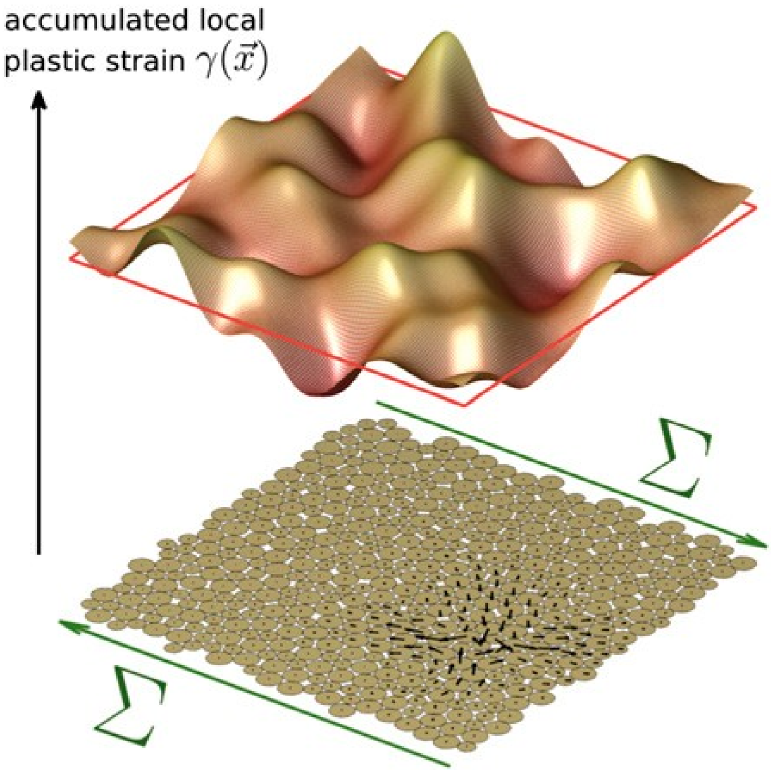
Scaling description of the yielding transition in soft amorphous solids at zero temperature

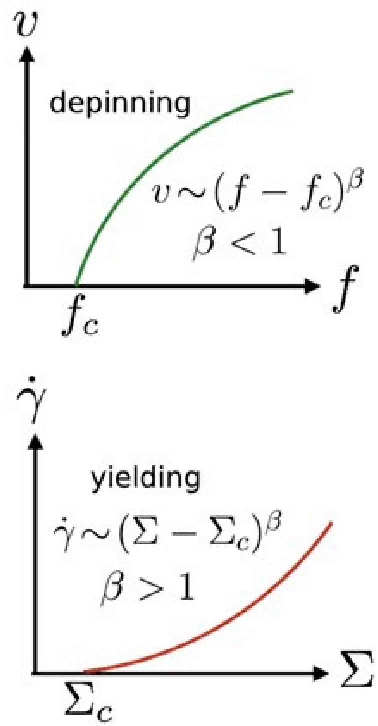
Jie Lin (林杰)[a], Edan Lerner[a], Alberto Rosso[b], and Matthieu Wyart[a,1]

[a]Center for Soft Matter Research, Department of Physics, New York University, New York, NY 10003; and [b]Laboratoire de Physique Théorique et Modèles Statistiques (Centre National de la Recherche Scientifique, Unité Mixte de Recherche 8626), Université de Paris-Sud, 91405 Orsay Cedex, France

Edited by David A. Weitz, Harvard University, Cambridge, MA, and approved August 21, 2014 (received for review April 8, 2014)

Similarity between the yielding transition to depinning ?





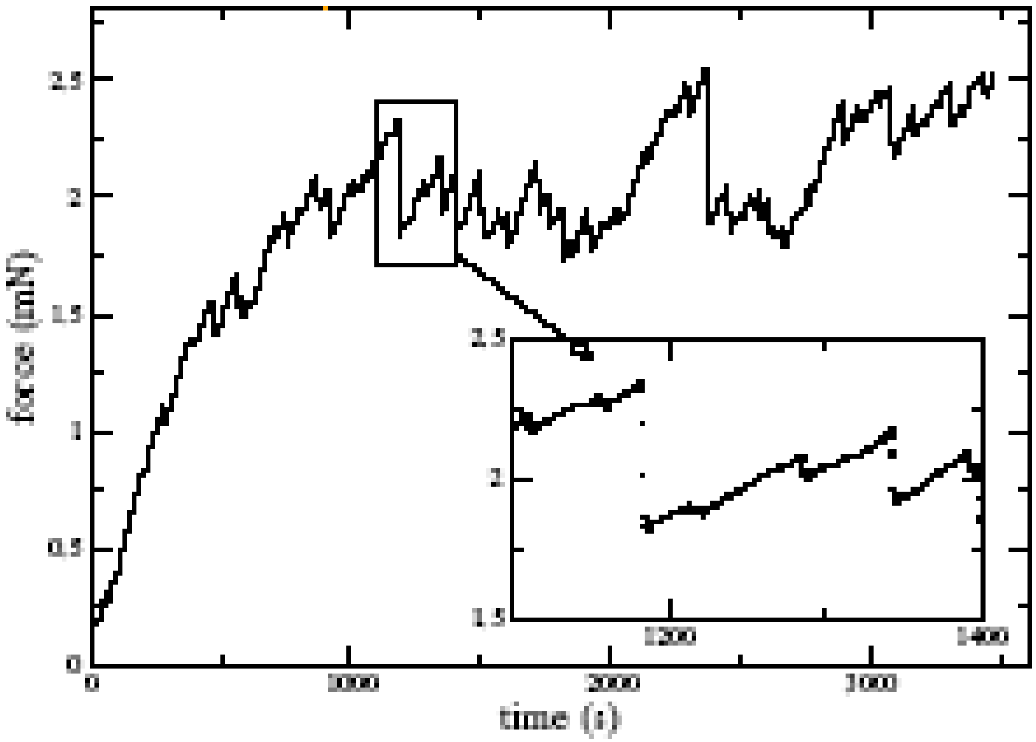
Lin et al, PNAS2014

Lin, Lerner, Rosso, Wyart, EPL 2014, PNAS 2014

Major difference with depinning problem: Kernel G(r-r') **is not positive everywhere**
⇒New universality class(es), different from depinning problems

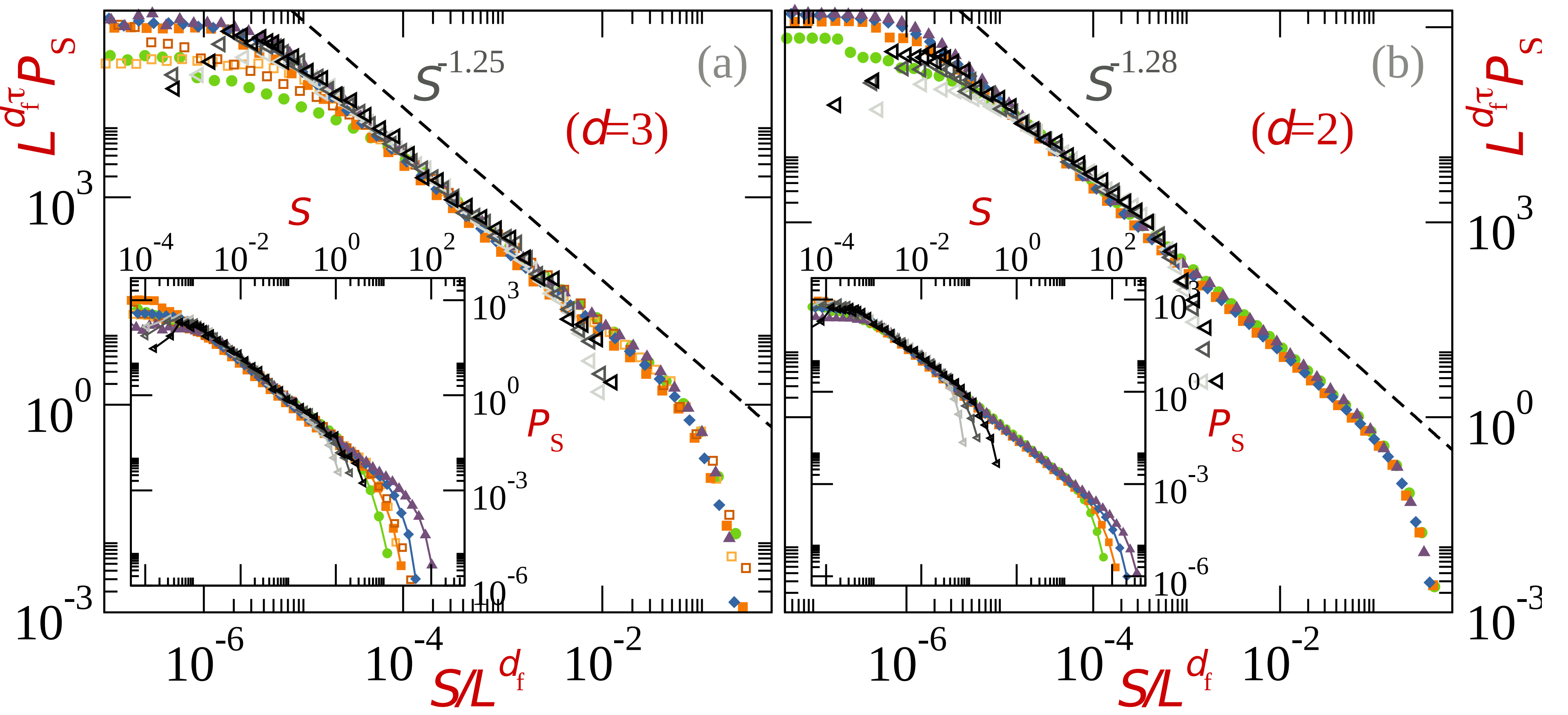
Avalanches: stress drops in the stress/strain curve.
Transition characterized by avalanche statistics (similar to Barkhausen noise, earthquakes) obtained from numerics/experiments.

Avalanche sizes display universal power laws in the limit of small strain rate (quasistatic)



« Earthquake like » statistics of stress drops

Avalanche sizes display universal power laws in the limit of small strain rate (quasistatic)



$$P(S) = S^{-\tau} P(S/L^{d_f})$$

Universal critical exponents, different from mean field

Exponent	τ	τ'	d_f	θ	γ
Expression	$P(S) \sim S^{-\tau}$	$P(T) \sim T^{-\tau'}$	$S_{\text{cut}} \sim L^{d_f}$	$p(x) = x^{\theta}$ with $x \equiv \sigma_y - \sigma$	$S \sim T^{\gamma}$
2D EPM					
(Talamali *et al.*, 2011) [spring coupling $k \to 0$]	1.25 ± 0.05	—	~ 1	—	—
(Budrikis and Zapperi, 2013) [spring coupling $k \gtrsim 0.1$]	1.364 ± 0.005	1.5 ± 0.09	$\gtrsim 1^{\dagger}$	—	~ 1.85
(Lin *et al.*, 2014b) [extremal]	~ 1.2	~ 1.6	$1.10 \pm 0.04^{*}$	~ 0.50	—
(Liu *et al.*, 2016) [$\dot{\gamma} \to 0$]	1.28 ± 0.05	1.41 ± 0.04	0.90 ± 0.07	0.52 ± 0.03	1.58 ± 0.07
(Budrikis *et al.*, 2017) [adiabatic loading]	1.280 ± 0.003	—	—	0.354 ± 0.004	1.8 ± 0.1
3D EPM					
(Lin *et al.*, 2014b) [extremal]	~ 1.3	~ 1.9	$1.50 \pm 0.05^{*}$	~ 0.28	—
(Liu *et al.*, 2016) [$\dot{\gamma} \to 0$]	1.25 ± 0.05	1.44 ± 0.04	1.3 ± 0.1	0.37 ± 0.05	1.58 ± 0.05
(Budrikis *et al.*, 2017) [adiabatic loading]	1.280 ± 0.003	—	—	0.354 ± 0.004	1.8 ± 0.1

No complete theory of critical exponents yet (in contrast to depinning)

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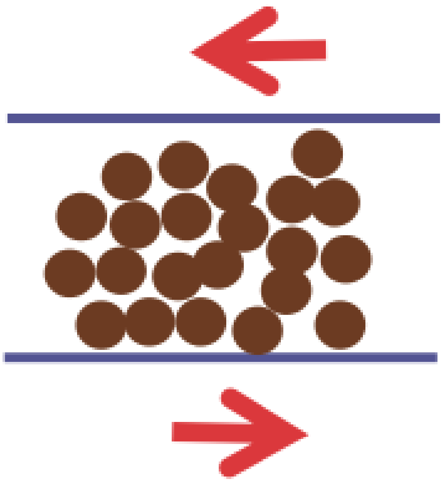
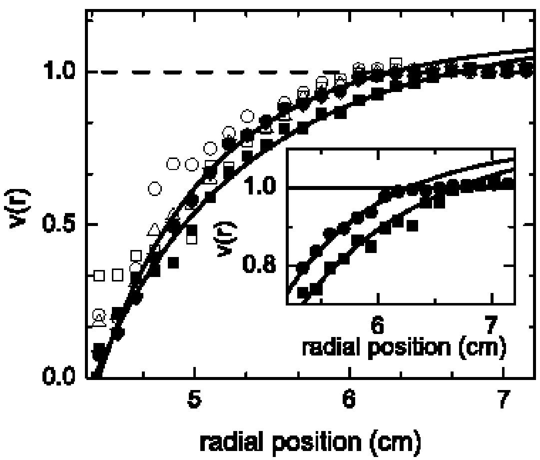


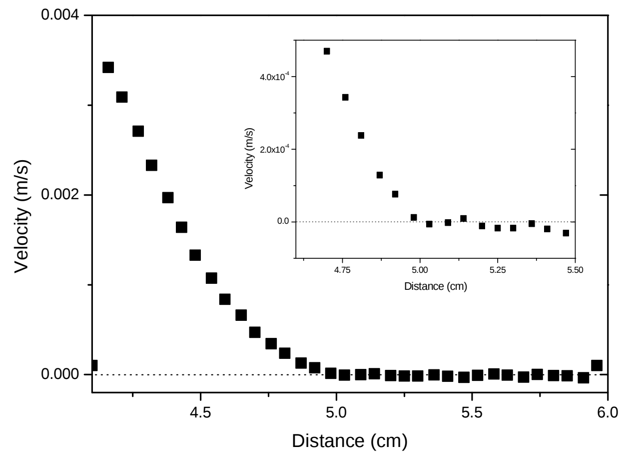
Yield (arrested->flow) transition as a first order dynamical transition: strain localisation / shear banding

Coexistence of flowing regions and solid regions at the same value of the stress

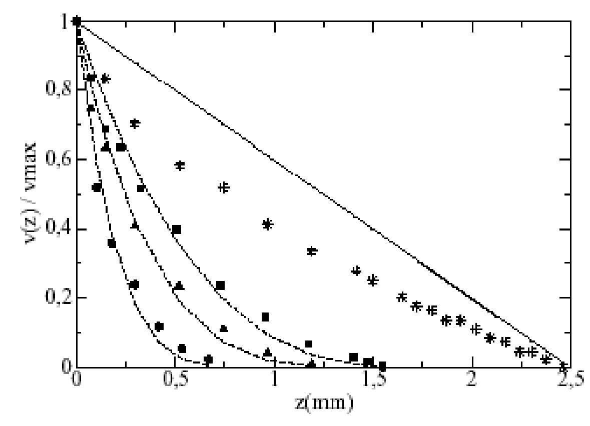
Bubble Rafts
(Dennin *et al.*, 2004)



Chocolate
(Coussot *et al.*)

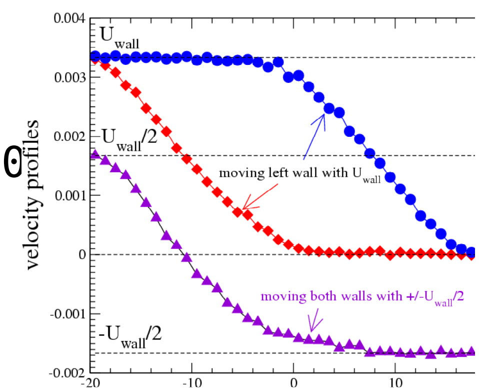


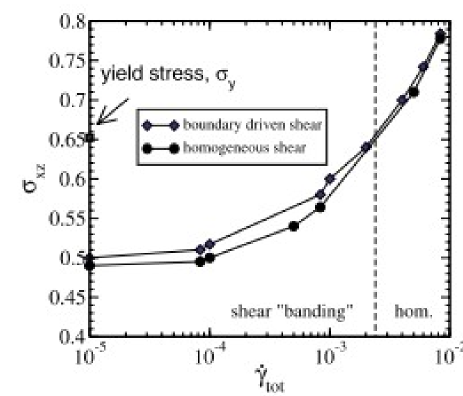
Granular pastes
(Barentin et al., 2003)



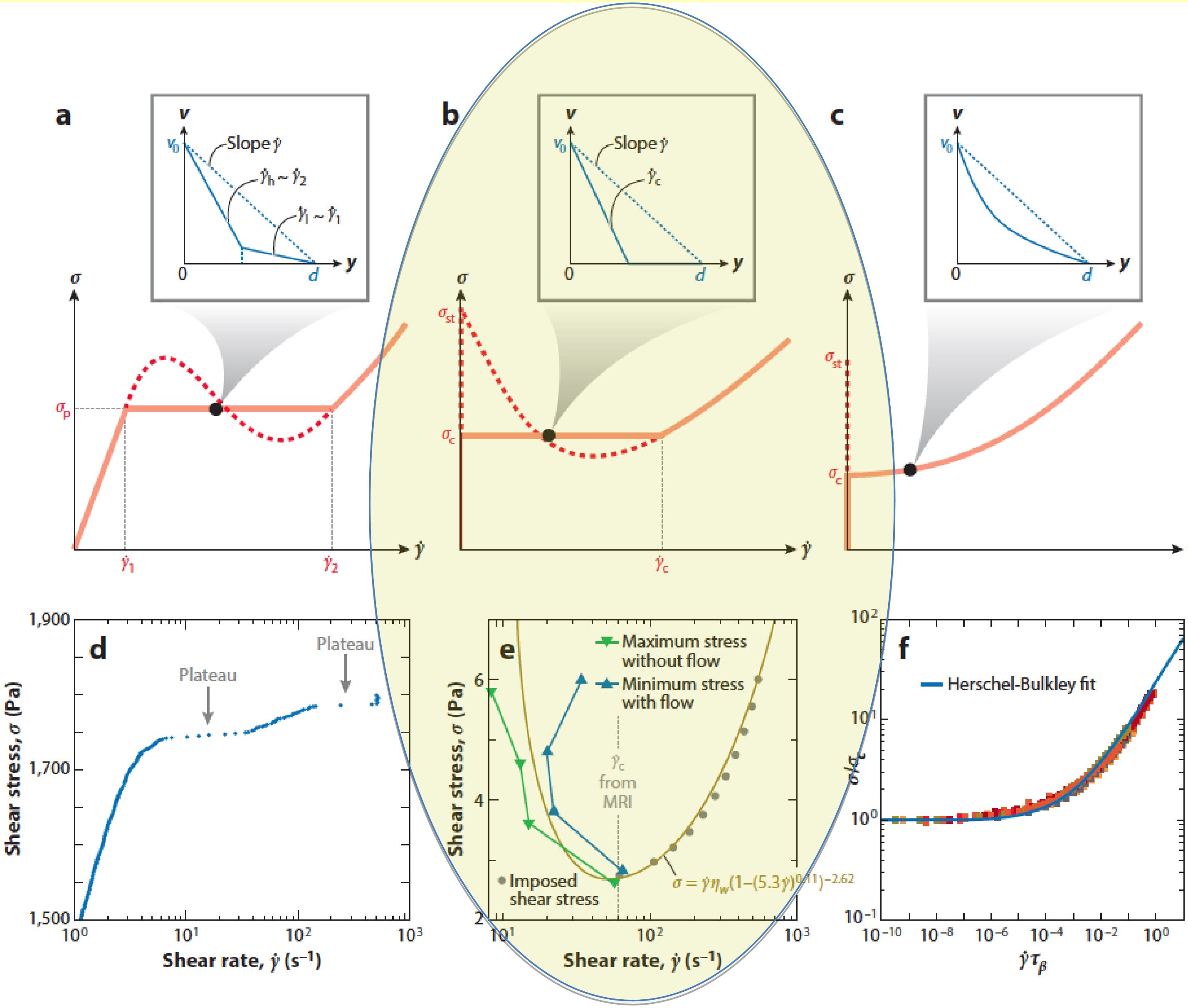
ennard-Jones glass
imulation, Varnik, Bocquet, JLB, 200

Explained » by static vs dynamic
eld stress





Strain localisation/ Shear banding



Flow profile (cylindrical Couette)

Flow curve

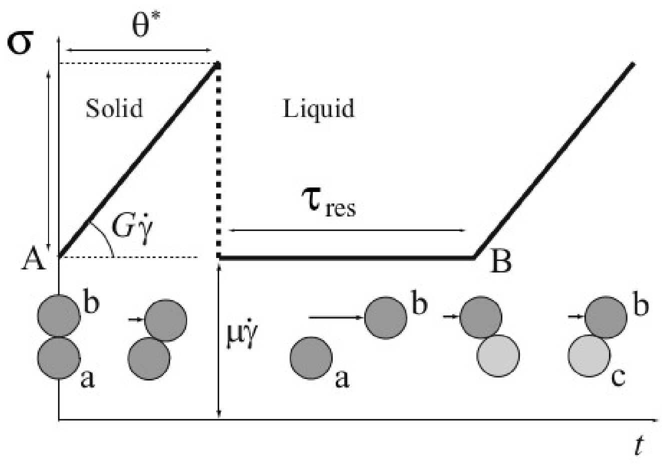
Example system

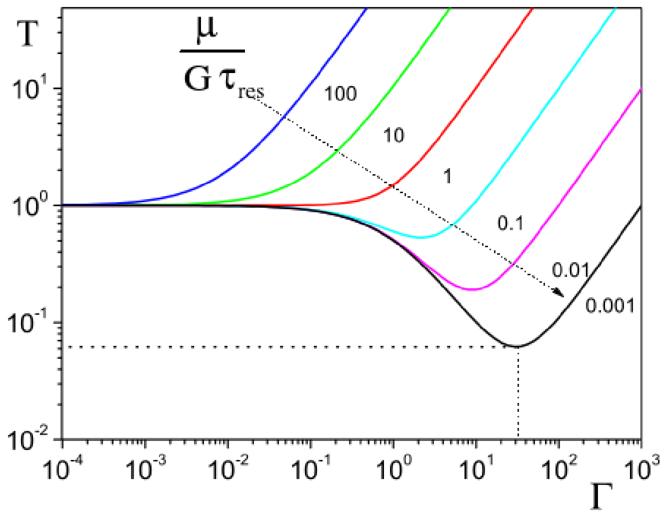
Divoux, Fardin, Manneville, Lerouge, *Annual review fluid mechanics* 2016

Strain localisation/ Shear banding

A possible mechanism from a mesoscopic viewpoint (Coussot and Ovarlez, Martens et al): long plastic events (large "healing time")

Coussot and Ovarlez mean field analysis (EPJE 2010)



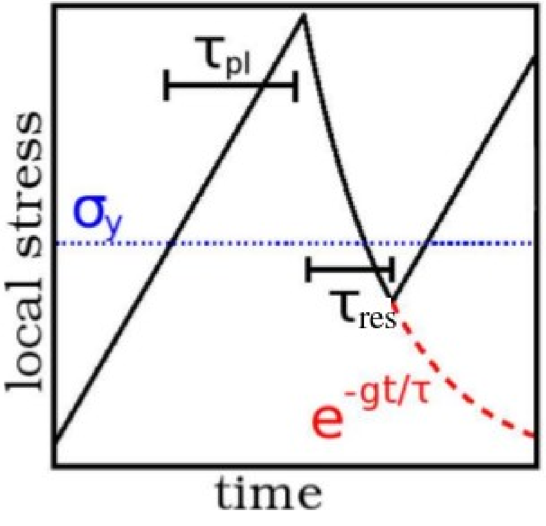


Constitutive curve becomes non monotonic at large τ_{res}

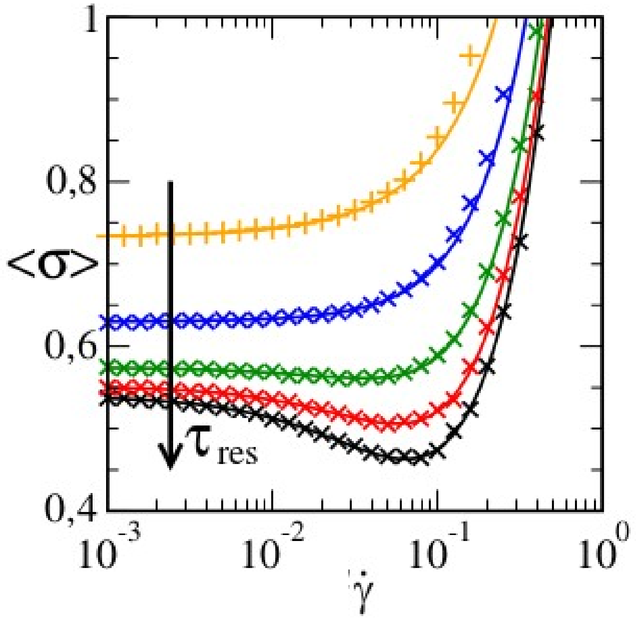
Strain localisation/ Shear banding

A possible mechanism from a mesoscopic viewpoint (Coussot and Ovarlez, Martens et al): long plastic events (large “healing time”)

Assembly of elastoplastic blocks interacting via elastic propagator.
Healing time τ_{res} before elastic recovery varies.



Life cycle of a single block



Flow curves

K Martens, L. Bocquet, JLB, Soft Matter 2012

Strain localisation/ Shear banding

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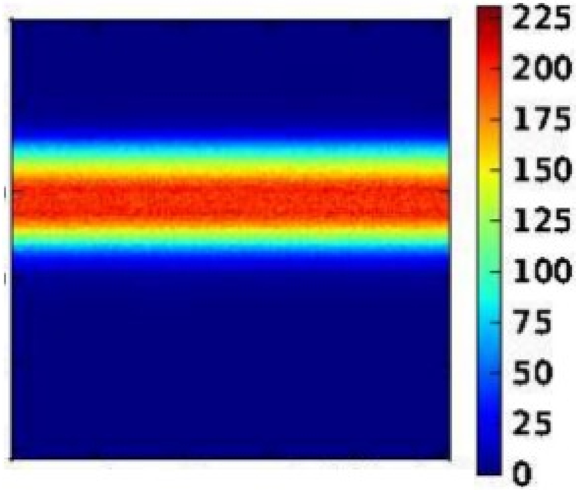
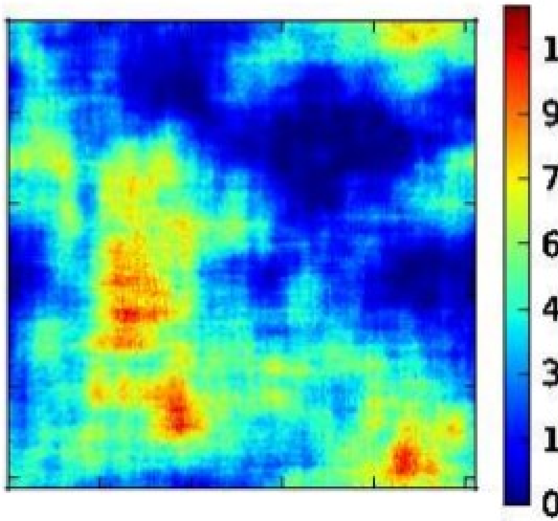


Cumulated plastic activity

Why linear structure ?

$$\widetilde{\sigma}_{el} = \widetilde{G}.\widetilde{\varepsilon}_p \quad =0$$

Outside an homogeneous plastic band
(soft mode of the elastic propagator)



Elastic propagator replaced by short range interaction

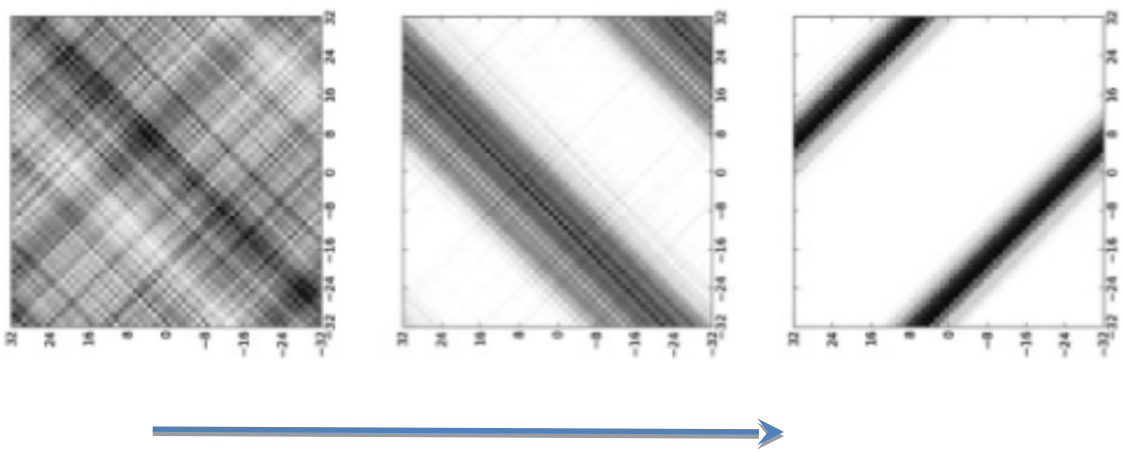
Martens, . Bocquet, JLB, Soft Matter 2012
Tyukodi, Patinet, Roux, Vandembroucq 2016 “soft modes in the depinning transition”

Strain localisation/ Shear banding

Other possibilities for shear banding :

- ageing (microscopic view, Shi and Falk, PRL 2007; mesoscopic, Vandembroucq and Roux, PRB 2011)

- Inertia (Nicolas et al, PRL 2016)

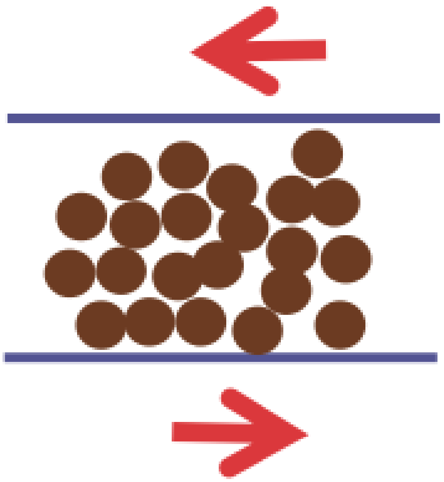


Age increases

Strain localisation takes place in aged systems (from Vandembroucq and Roux)

Outline

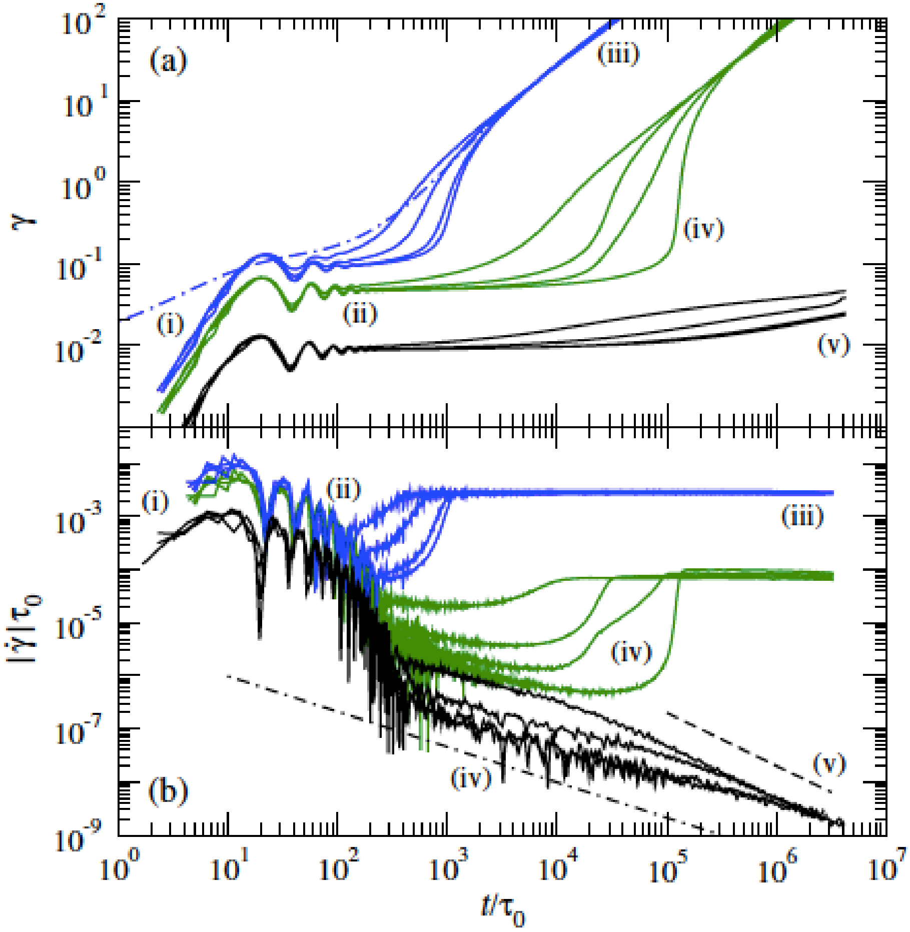
- Flow of soft amorphous solids
- Elastoplastic models of flow
- Mean field analysis
- Second order transition -Avalanches
- First order transition –Strain localisation
- Transients - Creep



Creep: Apply a fixed stress σ and measure the strain γ (t)

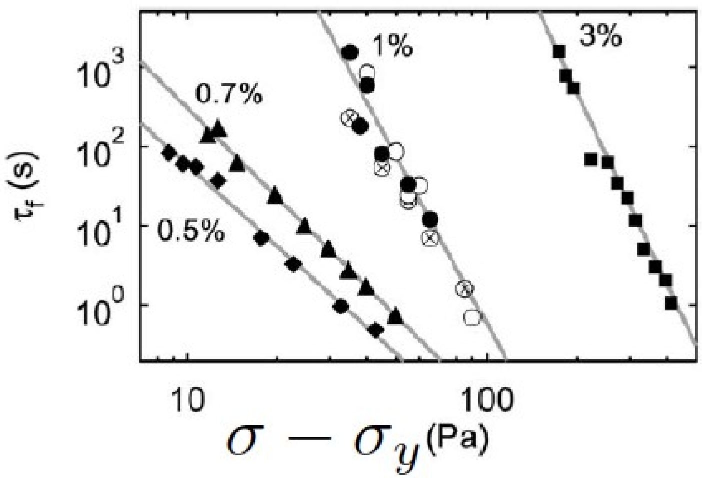
Siegenbürger et al, PRL 2012
Creep in a colloidal glass

Strain response: different stress levels, different waiting times



Divoux et al, Soft Matter 2011
Carbopol microgel

Fluidization time behaves as a power law of the distance to yield stress



Stress controlled version of elastoplastic models

Chen Liu, Kirsten Martens, JLB

Mean field version: arxiv:1705.06912; Phys rev lett. 2017

Spatial version: arXiv:1807.02497 Soft matter 2018

$$\partial_t P(\sigma,t) = -G_o\dot{\gamma}(t)\partial_\sigma P + \alpha\Gamma(t)\partial_\sigma^2 P - \frac{1}{\tau}\theta(|\sigma|-\sigma_c)P + \Gamma(t)\delta(\sigma)$$

Elastic Response To Shear — Mechanical Noise From Plastic Events — Plastic events & Recovery

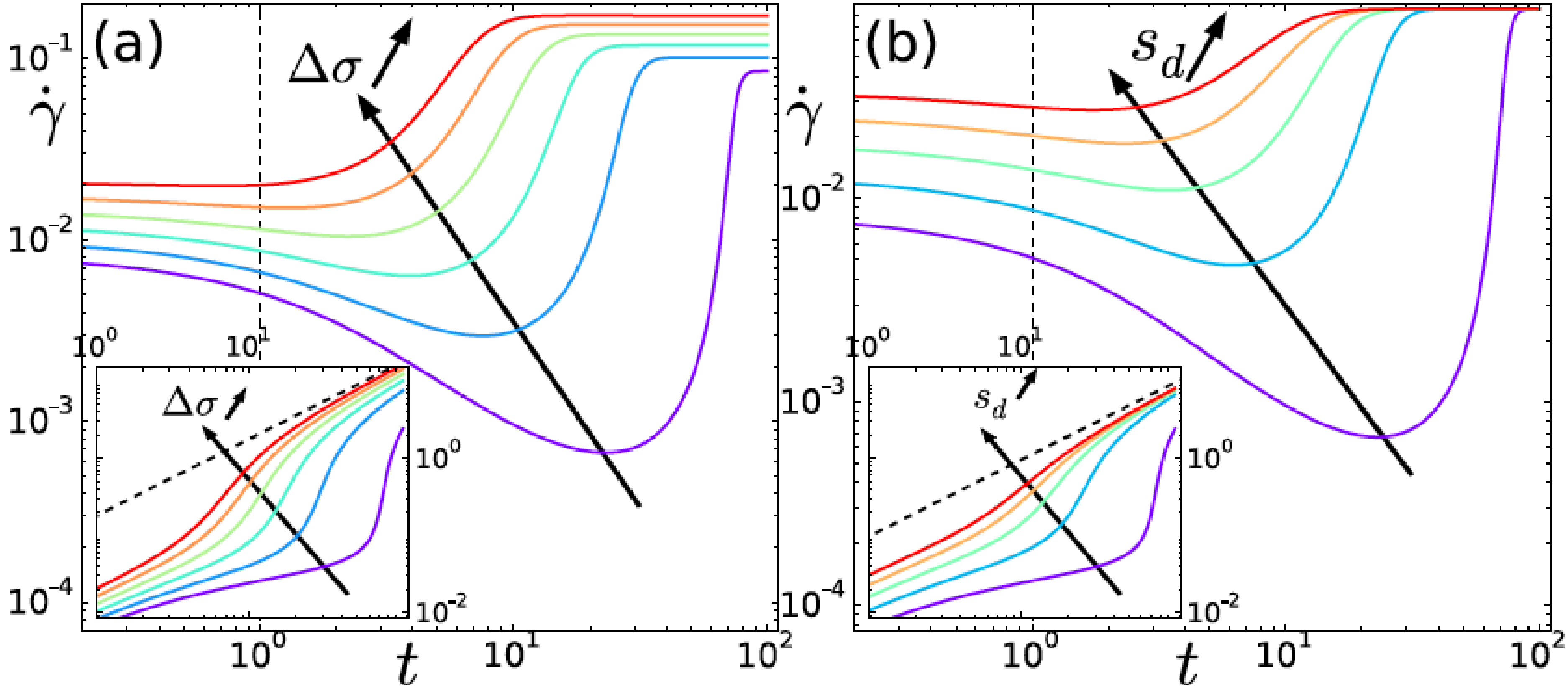
$$\Gamma(t) = \frac{1}{\tau}\int \theta(|\sigma|-\sigma_c)P(\sigma,t)d\sigma$$ Plastic Events Over All System

By imposing $\dot{\gamma}(t) = Cst$ → Steady State Shear Rheology

By imposing $\int P(\sigma)\sigma d\sigma = Cst$ → Stress control protocol: Creep

$$\dot{\gamma}(t) = \frac{1}{\tau G_o}\int_{|\sigma|>\sigma_c} P(\sigma,t)\sigma d\sigma$$

Results qualitatively similar to experiments; very strong dependence on the initial condition for the probability distribution function

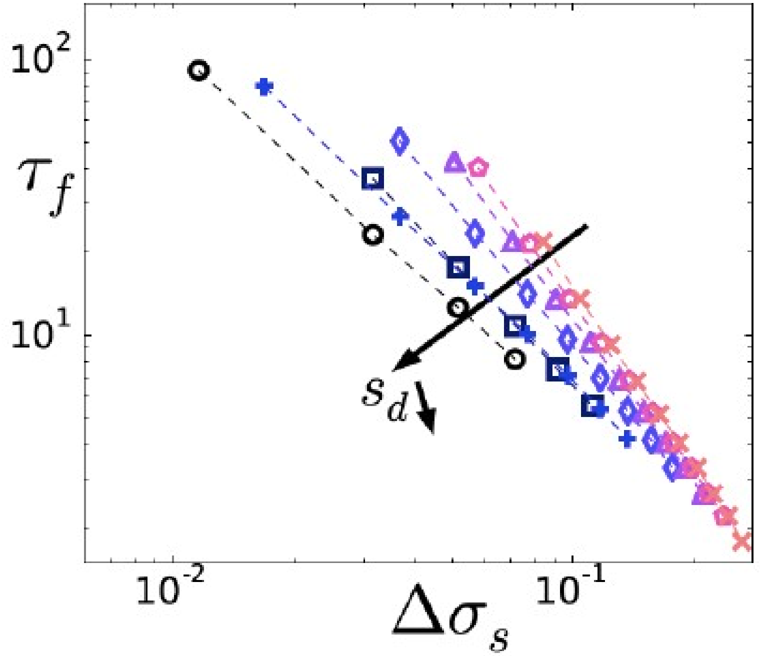


$$\Delta\sigma = \sigma^{imp} - \sigma_y(\alpha)$$

S_d: decreases when system ages

Fluidization time follows a power law with the static yield stress as a reference (can be identified with overshoot in stress strain curve).

Exponent is not universal, depends on system age.



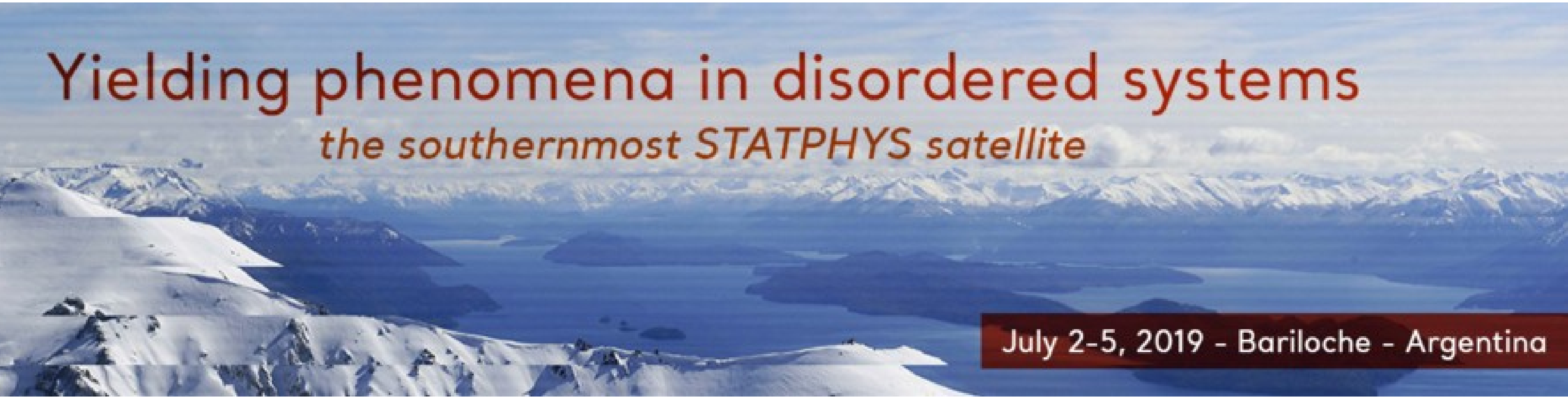
$$\Delta\sigma_s = (\sigma - \sigma_y^S)$$

In spatially resolved models, fluidisation is announced by a strong cooperativity in plastic activity – precursor of rupture (see ***arXiv:1807.02497)***

Conclusions

- Flow of soft solids can be described by simple elasto plastic models.
- Second « order » dynamical phase transition with similarities to depinning (avalanches, critical exponents) but different universality class.
- Possibility of first order transition (strain localisation…), several different mechanisms
- Non universal creep behaviour
- Perspectives: living tissues, temperature effects, critical behaviour…

Review paper : Review of Modern Physics 2018



A Satellite meeting of :

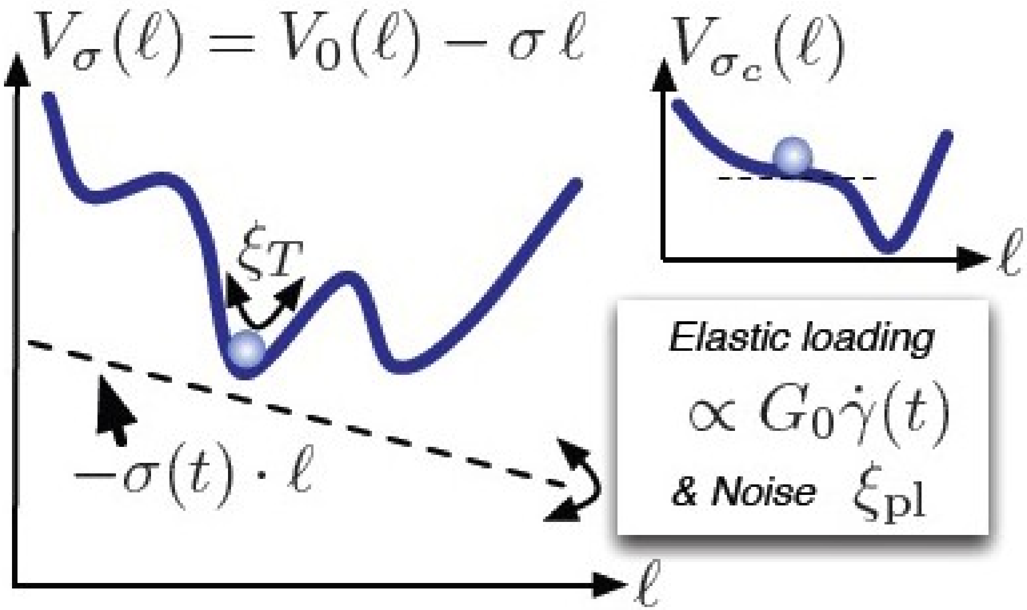
StatPhys 27

Buenos Aires, 8th-12th July, 2019

https://statphys27.df.uba.ar/
https://yielding2019.sciencesconf.org/

Note: mechanical noise is different from thermal noise!
(different from ''soft glassy rheology'' picture)

Potential Energy Landscape
Picture for a small region (STZ):



- Thermal noise acts on strain variable l in a fixed landscape biased by the stress
- Mechanical noise acts a diffusive process on the stress bias itself

=> Very different escape times (Arrhenius vs diffusive)

A. Nicolas, K. Martens, JLB, EPL 2014
E. Agoritsas et al, EPJ E 2015

Understanding gained from studies of elastoplastic models:

- Mean field analysis: Hébraud-Lequeux 1998, Herschel Bulkley law with exponent 1/2

- Acceleration of diffusion by active deformation (Martens, Bocquet, JLB, PRL 2011)

- Interplay between aging and strain localisation (Vandembroucq and Roux, PRB 2011)

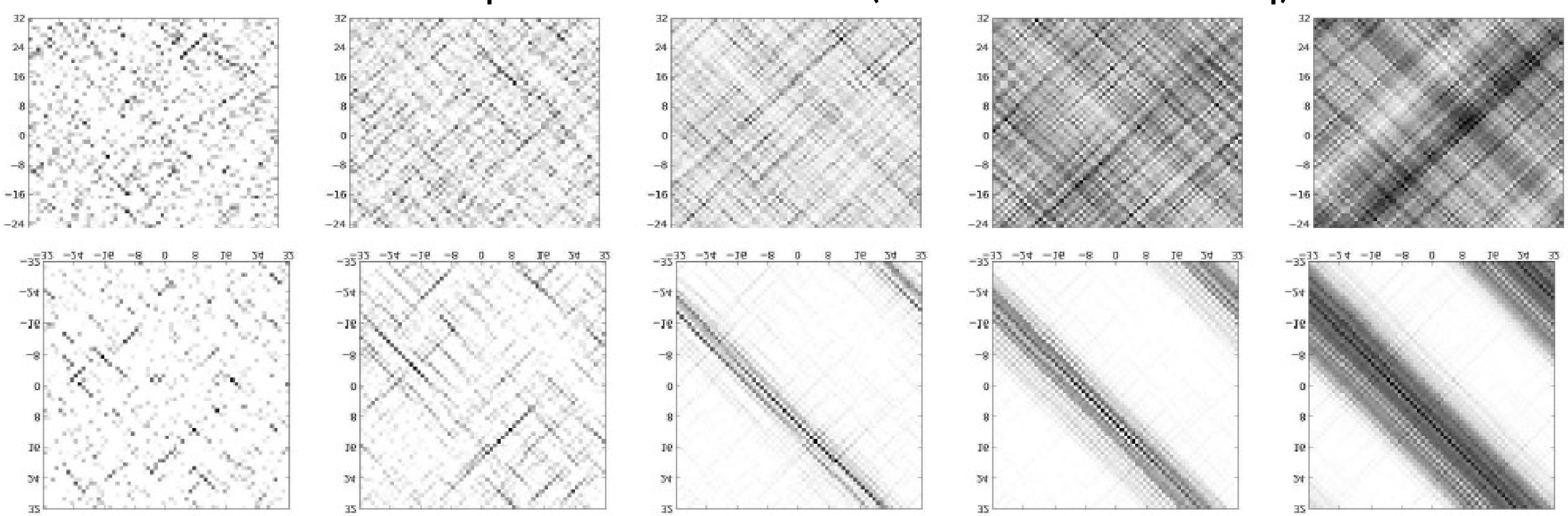
- criteria for strain localisation in soft materials (Martens, Bocquet, JLB, Soft Matter 2012)

- Introducing thermal activation of local events allows one to reproduce « compressed exponential » relaxation (Ferrero, Martens, JLB, PRL 2014).

Accumulated plastic deformation (Roux Vandembroucq)

As quenched

Aged



Mesoscopic description- an old idea

Self-organized criticality in a crack-propagation model of earthquakes

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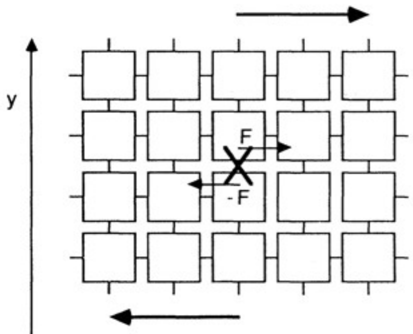
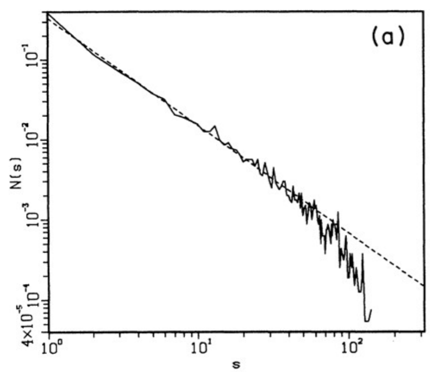
(Received 14 August 1990)

Phys Rev A, 1991

Spring network with threshold in force



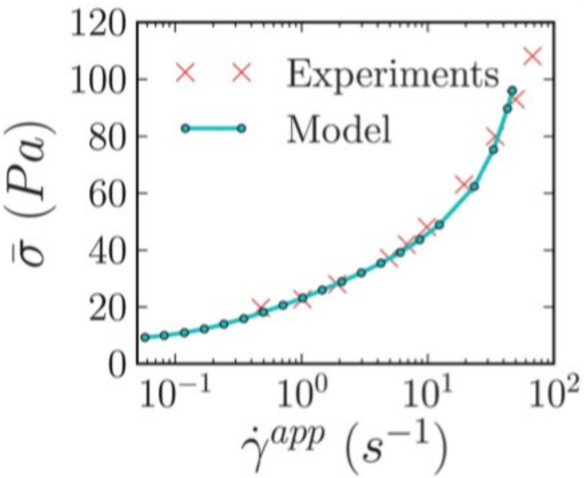


external stress field. When the stress somewhere exceeds a critical value (which is must be eventually since the stress is ever increasing), the shear stress is released while the medium undergoes a local shear deformation (rupture). This causes a very anisotropic redistribution of elastic forces, falling off roughly as $1/r^d$ with the distance from the instability:[18] Somewhere the shear force in-

Slope -1.4 in 2D

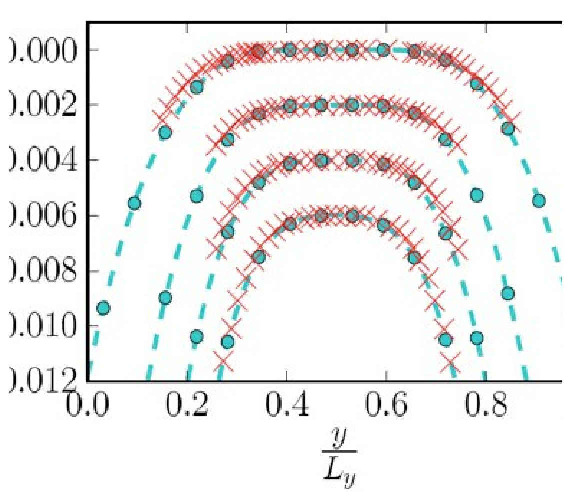
Validation of the approach

Model can reproduce quantitatively average flow and fluctuations of a dense suspension in microchannel geometries (experiments: Goyon et al, 2011; Nicolas and Barrat, PRL 2013) – Parameters calibrated on homogeneous flow curve.

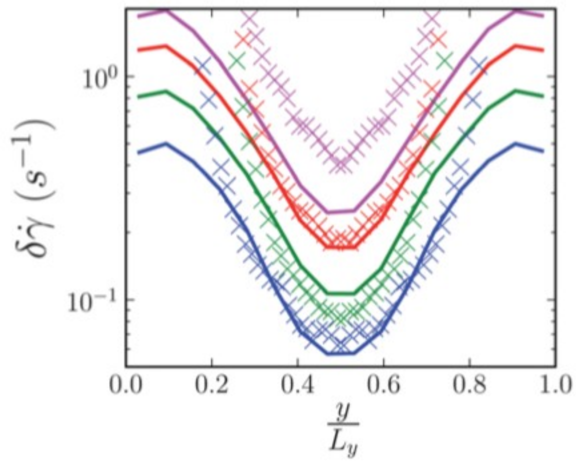


$$\sigma = \sigma_Y + A\dot{\gamma}^{1/\beta}$$

Herschel Bulkley equation



Velocity profile



Velocity fluctuations

Flow of amorphous solids:

- Statistical physics problem: elementary events identified, space time interactions and correlations between events lead to avalanches and noise.
- Jamming/Yielding (from flowing suspension to a solid paste) has features of a dynamical phase transition (depinning ?)

$$\sigma = \sigma_Y + A\dot{\gamma}^{1/\beta} \longleftrightarrow \dot{\gamma} \propto (\sigma - \sigma_{\mathrm{yield}})^{\beta}$$

- Universal features from granular media to metallic glasses
- Different (simpler ?) from extensively studied crystalline plasticity which involves topological flow defects (dislocations).

=> Gain qualitative understanding of phenomena at different scales with simple models, and tune model parameters for quantitative predictions.